1996 American Junior High School Mathematics Exam (AJHSME) Answers

1. B
2. C
3. A
4. B
5. A
6. C
7. B
8. B
9. D
10. D
11. D
12. B
13. E
14. B
15. E
16. C
17. C
18. A
19. C
20. A
21. D
22. B
23. E
24. C
25. A

1. The factors of 36 are $1, 2, 3, 4, 6, 9, 12, 18,$ and 36.

The multiples of 4 up to 36 are $4, 8, 12, 16, 20, 24, 28, 32$ and 36.

Only $4, 12$ and 36 appear on both lists, so the answer is 3, which is option $\boxed{B}$.

2. Jose gets $10 - 1 = 9$, then $9 \cdot 2 = 18$, then $18 + 2 = 20$.

Thuy gets $10 \cdot 2 = 20$, then $20 - 1 = 19$, and then $19 + 2 = 21$.

Kareem gets $10 - 1 = 9$, then $9 + 2 = 11$, and then $11 \cdot 2 = 22$.

Thus, Kareem gets the highest number, and the answer is $\boxed{C}$.

3. Obviously 1 is in the top left corner, 8 is in the top right corner, and 64 is in the bottom right corner. To find the bottom left corner, subtract 7 from 64 which is 57. Adding the results gives $1 + 8 + 57 + 64 = 130$ which is answer $\boxed{A}$.

4. Solution1) First, notice that each number in the numerator is a multiple of 2, and each number in the denominator is a multiple of 3. This suggests that each expression can be factored. Factoring gives:

$$\frac{2(1 + 2 + 3 + \cdots + 17)}{3(1 + 2 + 3 + \cdots + 17)}$$

Since all the material in the parentheses is the same, the common factor in the numerator and the denominator may be cancelled, leaving $\frac{2}{3}$, which is option $\boxed{B}$.

Solution2) There are 17 terms in the numerator $2 + 4 + 6 + \ldots + 30 + 32 + 34$.
Consider adding those terms, but in a different order. Start with the last two terms, $2 + 34$, and then add the next two terms on the outside, $4 + 32$, and continue.

You will get 8 pairs of numbers that add to 36, while the 9^{th} number in the middle will be alone. That number is 18. Adding all the numbers gives $8 \cdot 36 + 18 = 306$.

Similarly, the denominator has 17 terms of $3 + 6 + 9 + \ldots + 45 + 48 + 51$. There are 8 pairs of numbers that add up to 54, with the 9^{th} number in the center being 27. The total of all the numbers is $8 \cdot 54 + 27 = 459$.

The answer is $\frac{306}{459}$. Eyeballing the options, the fraction is clearly under 1, but more than $\frac{1}{2}$. Thus, the answer must be $\frac{2}{3}$, or $\boxed{B}$. Alternately, you can do the work by factoring out a 9 in the numerator to give $\frac{34}{51}$. Factoring out 17 will give the desired answer.

Solution3) Start by finding a pattern:

$$\frac{2}{3} = \frac{2}{3}$$

$$\frac{2+4}{3+6} = \frac{6}{9} = \frac{2}{3}$$

$$\frac{2+4+6}{3+6+9} = \frac{12}{18} = \frac{2}{3}$$

Each step doesn't seem to change the value of the fraction, so $\boxed{B}$ is the right answer.

5. Solution1) First, note that $P < Q < 0 < R < S < T$. Thus, P and Q are negative, while R, S, and T are positive.

Option A is the difference of two numbers. If the first number is lower than the second number, the answer will be negative. In this case, $P < Q$, so $P - Q < 0$, and $P - Q$ is thus indeed a negative number. So option $\boxed{A}$ is correct.

Option B is the product of two negatives, which is always positive.

Option C starts with the quotient of a positive and a negative. This is negative. But this negative quotient is then multiplied by a negative number, which gives a positive number.

Option D has the product of two negatives in the denominator. Thus, the denominator is positive. Additionally, the numerator is (barely) positive. A positive over a positive will be positive.

Option E has the sum of two positives in the numerator. That will be positive. The denominator is also positive. Again, the quotient of two positives is positive.

Solution2) Estimate the numbers, and calculate.

$P \approx -3.5$

$Q \approx -1.1$

$R \approx 0.1$

$S \approx 0.9$

$T \approx 1.5$

For option A, the value of $P - Q = -3.5 - (-1.1) = -3.5 + 1.1 = -2.4$. Thus $\boxed{A}$ is the right answer.

For option B, $PQ = -3.5 \cdot -1.1 = +3.85$

For option C, $\frac{S}{Q} \cdot P = \frac{0.9}{-1.1} \cdot -3.5 \approx -0.818 \cdot -3.5 \approx +2.836$

For option D, $\frac{R}{PQ} = \frac{0.1}{3.85} \approx +0.026$

For option E, $\frac{S+T}{R} = \frac{0.9+1.5}{0.1} = \frac{2.4}{0.1} = +24$

6. Since we want the smallest possible result, and we are only adding and multiplying positive numbers over, we can "prune" the set to the three smallest numbers $\{3, 5, 7\}$. Using bigger numbers will create bigger sums and bigger products.

From there, compute the 3 ways you can do the two operations:

$(3 + 5)7 = 8 \cdot 7 = 54$

$(3 + 7)5 = 10 \cdot 5 = 50$

$(7 + 5)3 = 12 \cdot 3 = 36$

The smallest number is 36, giving an answer of $\boxed{C}$

7. Call this month "Month 0". Make a table of the fish that Brent and Gretel have each month.

$\text{Month / Brent / Gretel}$

$0 / 4 / 128$

$1 / 16 / 256$

$2 / 64 / 512$

$3 / 256 / 1024$

$4 / 1024 / 2048$

$5 / 4096 / 4096$

You could create a similar table without doing all of the calculations by converting all the goldfish into powers of 2. In this table, you could increase Brent's goldfish by two powers of 2, while increasing Greta's fish by one power of 2.

Either way, in 5 months they will have the same number of fish, giving an answer of $\boxed{B}$

8. If $AB = 10$ and $BC = 4$, then $(10-4) \leq AC \leq (10+4)$ by the triangle inequality. In the triangle inequality, the equality is only reached when the "triangle" ABC is really a degenerate triangle, and ABC are collinear.

Simplifying, this means the smallest value AC can be is 6.

Applying the triangle inequality on ACD with $AC = 6$ and $CD = 3$, we know that $6-3 \leq AD \leq 6+3$ when AC is minimized. If AC were larger, then AD could be larger, but we want the smallest AD possible, and not the largest. Thus, AD must be at least 3, but cannot be smaller than 3. Therefore, $\boxed{B}$ is the answer.

This answer comes when $ABCD$ are all on a line, with $A = 0, B = 10, C = 6,$ and $D = 3$.

9. If 5 times a number is 2, then $5x = 2 \rightarrow x = \frac{2}{5}$, and the number is $\frac{2}{5}$.

If the number is $\frac{2}{5}$, then 100 times its reciprocal is 100 times $\frac{5}{2}$, which is 250, giving an answer of $\boxed{D}$.

10. The tank started at $\frac{1}{8}$ full, and ended at $\frac{5}{8}$ full. Therefore, Walter filled $\frac{5}{8} - \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$ of the tank.

If Walter fills half the tank with 7.5 gallons, then Walter can fill two halves of the tank (or a whole tank) with $7.5 \times 2 = 15$ gallons, giving an answer of $\boxed{D}$

11. Estimate each of the options.

A will give a number that is just over 3.

B will give a number that is just under 3. This eliminates B, because A is bigger.

C will give a number that is barely over 0, since it is three times a tiny number. This eliminates C, because A is bigger.

D will give a huge number. $\frac{1}{x}$ will get very, very large in magnitude when x gets close to zero. You can see this by examining the sequence $x = 0.1$, which gives 10 as the reciprocal, $x = 0.01$, which gives 100 as the reciprocal, and $x = 0.001$, which gives 1000 as the reciprocal. Thus, D will be huge, and this eliminates A.

E will give a small number, since you're dividing a tiny number into thirds. This eliminates E, and gives $\boxed{D}$ as the answer.

12. Solution1) Adding all of the numbers gives us $\frac{11 \cdot 12}{2} = 66$ as the current total. Since there are 11 numbers, the current average is $\frac{66}{11} = 6$. We need to take away a number from the total and then divide the result by 10 because there will only be 10 numbers left to give an average of 6.1. Setting up the equation:

$$\frac{66 - x}{10} = 6.1$$

$$66 - x = 61$$

$$x = 5$$

Thus, the answer is $\boxed{B}$

Solution2) Similar to the first solution, the current total is 66. Since there are 11 numbers on the list, taking 1 number away will leave 10 numbers. If those 10numbers have an average of 6.1, then those 10 numbers must have a sum of $10 \times 6.1 = 61$. Thus, the number that was removed must be $66 - 61 = 5$, and the answer is $\boxed{B}$.

13. Solution1) If the participation increases by 50%, then it is the same as saying participation is multipled by a factor of $100\% + 50\% = 1 + 0.5 = 1.5$.

In 1997, participation will be $800 \cdot 1.5 = 1200$.

In 1998, participation will be $1200 \cdot 1.5 = 1800$

In 1999, participation will be $1800 \cdot 1.5 = 2700$, giving an answer of $\boxed{E}$.

Solution2) Since the percentage increase is the same each year, this is an example of exponential growth with a base of 1.5. In three years, there will be $1.5^3 = \frac{27}{8}$ times as many participants. Multiplying this by the 800 current participants, there are 2700 participants, and the answer is $\boxed{E}$.

14. Looking at the vertical column, the three numbers sum to 23. If we make the numbers on either end 9 and 8 in some order, the middle number will be 6. This is the minimum for the intersection.

Looking at the horizontal row, the four numbers sum to 12. If we minimize the three numbers on the right to 123, the first number has a maximum value of 6. This is the maximum for the intersection

Thus, the minimum of the intersection is 6, and the maximum of the intersection is 6. This means the intersection must be 6, and the other numbers must be 9 and 8 in the column, and 123 in the row. The sum of all the numbers is $12 + 23 - 6 = 29$, and the answer is $\boxed{B}$

15. To determine a remainder when a number is divided by 5, you only need to look at the last digit. If the last digit is 0 or 5, the remainder is 0. If the last digit is 1 or 6, the remainder is 1, and so on.

To determine the last digit of $1492 \cdot 1776 \cdot 1812 \cdot 1996$, you only need to look at the last digit of each number in the product. Thus, we compute $2 \cdot 6 \cdot 2 \cdot 6 = 12^2 = 144$. The last digit of the number $1492 \cdot 1776 \cdot 1812 \cdot 1996$ is also 4, and thus the remainder when the number is divided by $5$is also 4, which gives an answer of $\boxed{E}$.

16. Put the numbers in groups of 4:

$$(1-2-3+4)+(5-6-7+8)+(9-10-11+12)+\cdots+(1993-1994-1995+1996)$$

The first group has a sum of 0.

The second group increases the two positive numbers on the end by 1, and decreases the two negative numbers in the middle by 1. Thus, the second group also has a sum of 0.

Continuing the pattern, every group has a sum of 0, and thus the entire sum is 0, giving an answer of $\boxed{C}$.

17. The area of $\square OPQR$ is $2^2 = 4$.

The area of $\triangle PQT$ is $\frac{1}{2}bh = \frac{1}{2} \cdot PT \cdot PQ$.

If we set the areas equal, the area of PQT is 4. Also, note that $PQ = 2$. Plugging those in, we get:

$$4 = \frac{1}{2} \cdot PT \cdot 2$$

$$PT = 4$$

If $PT = 4$, and $PO = 2$, then $OT = 2$, and T must be 2 units to the left of the origin. This would be $(-2,0)$, giving answer $\boxed{C}$.

18. In June, Ana's pay is $2000 \cdot 1.2 = 2400$

In July, Ana's pay is $2400 \cdot 0.8 = 1920$, giving an answer of $\boxed{A}$.

19. In the first school, $2000 \cdot 22\% = 2000 \cdot 0.22 = 440$ students prefer tennis.

In the second school, $2500 \cdot 40\% = 2500 \cdot 0.40 = 1000$ students prefer tennis.

In total, $440 + 1000 = 1440$ students prefer tennis out of a total of $2000 + 2500 = 4500$ students

This means $\frac{1440}{4500} \cdot 100\% = \frac{32}{100} \cdot 100\% = 32\%$ of the students in both schools prefer tennis, giving answer $\boxed{C}$.

20. We look for a pattern, hoping this sequence either settles down to one number, or that it forms a cycle that repeats.

After 1 press, the calculator displays $\frac{1}{1-5} = -\frac{1}{4}$

After 2 presses, the calculator displays $\frac{1}{1 - -\frac{1}{4}} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$

After 3 presses, the calculator displays $\frac{1}{1 - \frac{4}{5}} = \frac{1}{\frac{1}{5}} = 5$

Thus, every three presses, the display will be 5. On press $3 \cdot 33 = 99$, the display will be 5. One more press will give $-\frac{1}{4}$, which is answer $\boxed{A}$.

21. To have an even sum with three numbers, we must add either $E + O + O$, or $E + E + E$, where O represents an odd number, and E represents an even number.

Since there are not three even numbers in the given set, $E+E+E$ is impossible. Thus, we must choose two odd numbers, and one even number.

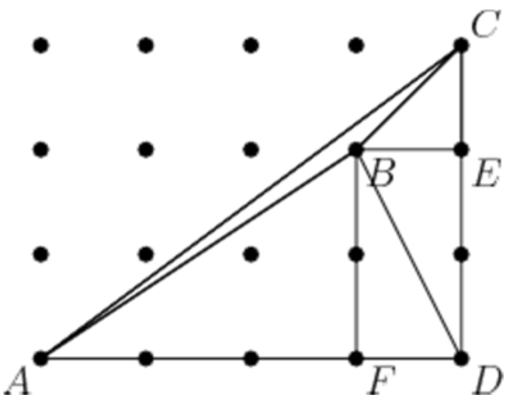
There are 2 choices for the even number.

There are 4 choices for the first odd number. There are 3 choices for the last odd number. But the order of picking these numbers doesn't matter, so this overcounts the pairs of odd numbers by a factor of 2. Thus, we have $\frac{4\cdot 3}{2}=6$ choices for a pair of odd numbers.

In total, there are 2 choices for an even number, and 6 choices for the odd numbers, giving a total of $2\cdot 6=12$ possible choices for a 3-element set that has an even sum. This is option $\boxed{D}$.

22.

Solution1)



$\triangle ADC$ takes up half of the 4x3 grid, so it has area of 6.

$\triangle ABD$ has height of $BF=2$ and a base of $AD=4$, for an area of $\frac{1}{2}\cdot 2\cdot 4=4$.

$\triangle CBD$ has height of $BE=1$ and a base of $CD=3$, for an area of $\frac{1}{2}\cdot 1\cdot 3=\frac{3}{2}$

Note that $\triangle ABC$ can be found by taking $\triangle ADC$, and subtracting off $\triangle ABD$ and $\triangle CBD$.

Thus, the area of $\triangle ABC = 6 - 4 - \frac{3}{2} = \frac{1}{2}$, and the answer is $\boxed{B}$.

There are other equivalent ways of dissecting the figure; right triangles $\triangle ABF, \triangle BCE$ and rectangle $\square BEDF$ can also be used.

Solution2) Using the Shoelace Theorem, and labelling the points $A(0, 0), B(3, 2), C(4, 3)$, we find the area is:

$(0, 0)$

$(3, 2)$

$(4, 3)$

$(0, 0)$

Area $\frac{1}{2} = |(3 \cdot 0 + 4 \cdot 2 + 0 \cdot 3) - (0 \cdot 2 + 3 \cdot 3 + 4 \cdot 0)| = \frac{1}{2}$ = , which is option $\boxed{B}$.

23. Solution1) Let p be the number of people in the company, and f be the amount of money in the fund.

The first sentence states that $50p = f + 5$

The second sentence states that $45p = f - 95$

Subtracing the second equation from the first, we get $5p = 100$, leading to $p = 20$

Plugging that number into the first equation gives $50 \cdot 20 = f + 5$, leading to $f = 995$, which is answer $\boxed{E}$.

Solution2) Since the company must employ a whole number of employees, the amount of money in the fund must be 5 dollars less than a multiple of 50. Only options A and E satisfy that requirement.

Additionally, the number must be 95 more than a multiple of 45. Since $45 \cdot 20 = 900$, the only number that is 95 more than a multiple of 45 out of options A and E is option $\boxed{E}$.

(Option B is also 95 more than a multiple of 45, but it was eliminated previously.)

24. Let $\angle CAD = \angle BAD = x$, and let $\angle ACD = \angle BCD = y$

From $\triangle ABC$, we know that $50 + 2x + 2y = 180$, leading to $x + y = 65$.

From $\triangle ADC$, we know that $x + y + \angle D = 180$. Plugging in $x + y = 65$, we get $\angle D = 180 - 65 = 115$, which is answer $\boxed{C}$.

25. Draw a circle with a radius of 2. Draw a concentric circle with radius 1. The edge of this inner circle is the set of all points that are 1 from the center, and 1 from the outer circle. In other words, it is the set of all points that are equidistant from the center of the circles to the outside of the big circle.

The inside of this circle of radius 1 is the set of all points that are closer to the center of the region than to the boundary of the outer circle. The "washer" region that is outside the circle of radius 1, but inside the circle of radius 2, is the set of all points that are closer to the boundary than to the center of the circle.

If you select a random point in a region of area B, the probability that the point is in a smaller subregion A is the ratio $\frac{A}{B}$. In this case, $B = \pi \cdot 2^2 = 4\pi$, and $A = \pi \cdot 1^2 = \pi$, and the ratio of areas is $\frac{\pi}{4\pi} = \frac{1}{4}$, and the answer is $\boxed{A}$.