MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

61st Annual

AMC 12 B

American Mathematics Contest 12B

Wednesday, February 24, 2010

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 28th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 16, 2010 or Wednesday, March 31, 2010. More details about the AIME and other information are on the back page of this test booklet.

The publication, reproduction or communication of the problems or solutions of the AMC 12 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of problems in paper or electronic form including posting on web-pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

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2010
AMC 12
CONTEST B

DO NOT OPEN UNTIL
WEDNESDAY, FEBRUARY 24, 2010

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 24, 2010. Nothing is needed from inside this package until February 24.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed First Class to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the competition rules.*

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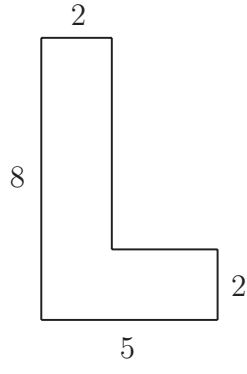
1. Makayla attended two meetings during her 9-hour work day. The first meeting took 45 minutes and the second meeting took twice as long. What percent of her work day was spent attending meetings?

   (A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

2. A big L is formed as shown. What is its area?



   (A) 22 **(B)** 24 **(C)** 26 **(D)** 28 **(E)** 30

3. A ticket to a school play costs x dollars, where x is a whole number. A group of 9th graders buys tickets costing a total of \$48, and a group of 10th graders buys tickets costing a total of \$64. How many values for x are possible?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

4. A month with 31 days has the same number of Mondays and Wednesdays. How many of the seven days of the week could be the first day of this month?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

5. Lucky Larry's teacher asked him to substitute numbers for a, b, c, d, and e in the expression $a - (b - (c - (d + e)))$ and evaluate the result. Larry ignored the parentheses but added and subtracted correctly and obtained the correct result by coincidence. The numbers Larry substituted for a, b, c, and d were $1, 2, 3$, and 4, respectively. What number did Larry substitute for e?

   (A) -5 **(B)** -3 **(C)** 0 **(D)** 3 **(E)** 5

6. At the beginning of the school year, 50% of all students in Mr. Wells' math class answered "Yes" to the question "Do you love math", and 50% answered "No." At the end of the school year, 70% answered "Yes" and 30% answered "No." Altogether, $x\%$ of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of x?

(A) 0 **(B)** 20 **(C)** 40 **(D)** 60 **(E)** 80

7. Shelby drives her scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today she drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 40 minutes. How many minutes did she drive in the rain?

(A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

8. Every high school in the city of Euclid sent a team of 3 students to a math contest. Each participant in the contest received a different score. Andrea's score was the median among all students, and hers was the highest score on her team. Andrea's teammates Beth and Carla placed 37th and 64th, respectively. How many schools are in the city?

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

9. Let n be the smallest positive integer such that n is divisible by 20, n^2 is a perfect cube, and n^3 is a perfect square. What is the number of digits of n?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

10. The average of the numbers $1, 2, 3, \ldots, 98, 99$, and x is $100x$. What is x?

(A) $\frac{49}{101}$ **(B)** $\frac{50}{101}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{51}{101}$ **(E)** $\frac{50}{99}$

11. A palindrome between 1000 and 10,000 is chosen at random. What is the probability that it is divisible by 7?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{7}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{5}$

12. For what value of x does

$$\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4 (x^2) + \log_8 (x^3) + \log_{16} (x^4) = 40\,?$$

(A) 8 **(B)** 16 **(C)** 32 **(D)** 256 **(E)** 1024

13. In $\triangle ABC$, $\cos(2A - B) + \sin(A + B) = 2$ and $AB = 4$. What is BC?

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** $2\sqrt{2}$ **(E)** $2\sqrt{3}$

14. Let a, b, c, d, and e be positive integers with $a + b + c + d + e = 2010$, and let M be the largest of the sums $a + b$, $b + c$, $c + d$ and $d + e$. What is the smallest possible value of M?

(A) 670 **(B)** 671 **(C)** 802 **(D)** 803 **(E)** 804

15. For how many ordered triples (x, y, z) of nonnegative integers less than 20 are there exactly two distinct elements in the set $\{i^x, (1 + i)^y, z\}$, where $i = \sqrt{-1}$?

(A) 149 **(B)** 205 **(C)** 215 **(D)** 225 **(E)** 235

16. Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1, 2, 3, \ldots, 2010\}$. What is the probability that $abc + ab + a$ is divisible by 3?

(A) $\frac{1}{3}$ **(B)** $\frac{29}{81}$ **(C)** $\frac{31}{81}$ **(D)** $\frac{11}{27}$ **(E)** $\frac{13}{27}$

17. The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?

(A) 18 **(B)** 24 **(C)** 36 **(D)** 42 **(E)** 60

18. A frog makes 3 jumps, each exactly 1 meter long. The directions of the jumps are chosen independently and at random. What is the probability that the frog's final position is no more than 1 meter from its starting position?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

19. A high school basketball game between the Raiders and the Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?

(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

20. A geometric sequence (a_n) has $a_1 = \sin x$, $a_2 = \cos x$, and $a_3 = \tan x$ for some real number x. For what value of n does $a_n = 1 + \cos x$?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

21. Let $a > 0$, and let $P(x)$ be a polynomial with integer coefficients such that

$$P(1) = P(3) = P(5) = P(7) = a, \text{ and}$$
$$P(2) = P(4) = P(6) = P(8) = -a.$$

What is the smallest possible value of a?

(A) 105 **(B)** 315 **(C)** 945 **(D)** 7! **(E)** 8!

22. Let $ABCD$ be a cyclic quadrilateral. The side lengths of $ABCD$ are distinct integers less than 15 such that $BC \cdot CD = AB \cdot DA$. What is the largest possible value of BD?

(A) $\sqrt{\frac{325}{2}}$ **(B)** $\sqrt{185}$ **(C)** $\sqrt{\frac{389}{2}}$ **(D)** $\sqrt{\frac{425}{2}}$ **(E)** $\sqrt{\frac{533}{2}}$

23. Monic quadratic polynomials $P(x)$ and $Q(x)$ have the property that $P(Q(x))$ has zeros at $x = -23, -21, -17$, and -15, and $Q(P(x))$ has zeros at $x = -59, -57, -51$, and -49. What is the sum of the minimum values of $P(x)$ and $Q(x)$?

(A) -100 **(B)** -82 **(C)** -73 **(D)** -64 **(E)** 0

24. The set of real numbers x for which

$$\frac{1}{x-2009} + \frac{1}{x-2010} + \frac{1}{x-2011} \geq 1$$

is the union of intervals of the form $a < x \leq b$. What is the sum of the lengths of these intervals?

(A) $\frac{1003}{335}$ **(B)** $\frac{1004}{335}$ **(C)** 3 **(D)** $\frac{403}{134}$ **(E)** $\frac{202}{67}$

25. For every integer $n \geq 2$, let $\text{pow}(n)$ be the largest power of the largest prime that divides n. For example $\text{pow}(144) = \text{pow}(2^4 \cdot 3^2) = 3^2$. What is the largest integer m such that 2010^m divides

$$\prod_{n=2}^{5300} \text{pow}(n)?$$

(A) 74 **(B)** 75 **(C)** 76 **(D)** 77 **(E)** 78

WRITE TO US!

Correspondence about the problems and solutions for this AMC 12 and orders for publications should be addressed to:

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2010 AIME

The 28th annual AIME will be held on Tuesday, March 16, with the alternate on Wednesday, March 31. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 1% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 39th Annual USA Mathematical Olympiad (USAMO) on April 27 - 28, 2010. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.unl.edu/amc