MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

11th Annual

AMC 10 B

American Mathematics Contest 10B

Wednesday, February 24, 2010

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 28th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 16, 2010 or Wednesday, March 31, 2010. More details about the AIME and other information are on the back page of this test booklet.

2010
AMC 10
CONTEST B

DO NOT OPEN UNTIL
WEDNESDAY, FEBRUARY 24, 2010

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 24, 2010. Nothing is needed from inside this package until February 24.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed First Class to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, e-mail, World Wide Web or media of any type is a violation of the competition rules.*

The American Mathematics Competitions

are Sponsored by

The Mathematical Association of America — MAA ..... www.maa.org/
The Akamai Foundation ..... www.akamai.com/

Contributors

Academy of Applied Sciences — AAS ..... www.aas-world.org
American Mathematical Association of Two-Year Colleges — AMATYC ..... www.amatyc.org
American Mathematical Society — AMS ..... www.ams.org
American Statistical Association — ASA ..... www.amstat.org
Art of Problem Solving — AoPS ..... www.artofproblemsolving.com
Awesome Math ..... www.awesomemath.org
Canada/USA Mathcamp — C/USA MC ..... www.mathcamp.org
Casualty Actuarial Society — CAS ..... www.casact.org
IDEA Math ..... www.ideamath.org
Institute for Operations Research and the Management Sciences — INFORMS ..... www.informs.org
MathPath ..... www.mathpath.org
Math Zoom Academy ..... www.mathzoom.org
Mu Alpha Theta — MAT ..... www.mualphatheta.org
National Council of Teachers of Mathematics — NCTM ..... www.nctm.org
Pi Mu Epsilon — PME ..... www.pme-math.org
Society of Actuaries — SOA ..... www.soa.org
U. S. A. Math Talent Search — USAMTS ..... www.usamts.org
W. H. Freeman and Company ..... www. whfreeman.com
Wolfram Research Inc. ..... www.wolfram.com

1. What is $100(100-3)-(100\cdot 100-3)$?

   (A) $-20{,}000$ **(B)** $-10{,}000$ **(C)** -297 **(D)** -6 **(E)** 0

2. Makayla attended two meetings during her 9-hour work day. The first meeting took 45 minutes and the second meeting took twice as long. What percent of her work day was spent attending meetings?

   (A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

3. A drawer contains red, green, blue and white socks with at least 2 of each color. What is the minimum number of socks that must be pulled from the drawer to guarantee a matching pair?

   (A) 3 **(B)** 4 **(C)** 5 **(D)** 8 **(E)** 9

4. For a real number x, define $\heartsuit(x)$ to be the average of x and x^2. What is $\heartsuit(1)+\heartsuit(2)+\heartsuit(3)$?

   (A) 3 **(B)** 6 **(C)** 10 **(D)** 12 **(E)** 20

5. A month with 31 days has the same number of Mondays and Wednesdays. How many of the seven days of the week could be the first day of this month?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

6. A circle is centered at O, $\overline{AB}$ is a diameter and C is a point on the circle with $\angle COB = 50^\circ$. What is the degree measure of $\angle CAB$?

   (A) 20 **(B)** 25 **(C)** 45 **(D)** 50 **(E)** 65

7. A triangle has side lengths 10, 10, and 12. A rectangle has width 4 and area equal to the area of the triangle. What is the perimeter of this rectangle?

   (A) 16 **(B)** 24 **(C)** 28 **(D)** 32 **(E)** 36

8. A ticket to a school play costs x dollars, where x is a whole number. A group of 9th graders buys tickets costing a total of \$48, and a group of 10th graders buys tickets costing a total of \$64. How many values for x are possible?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

9. Lucky Larry's teacher asked him to substitute numbers for a, b, c, d, and e in the expression $a - (b - (c - (d + e)))$ and evaluate the result. Larry ignored the parentheses but added and subtracted correctly and obtained the correct result by coincidence. The numbers Larry substituted for a, b, c, and d were $1, 2, 3$, and 4, respectively. What number did Larry substitute for e?

(A) -5 **(B)** -3 **(C)** 0 **(D)** 3 **(E)** 5

10. Shelby drives her scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today she drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 40 minutes. How many minutes did she drive in the rain?

(A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

11. A shopper plans to purchase an item that has a listed price greater than \$100 and can use any one of three coupons. Coupon A gives 15% off the listed price, Coupon B gives \$30 off the listed price, and Coupon C gives 25% off the amount by which the listed price exceeds \$100.

Let x and y be the smallest and largest prices, respectively, for which Coupon A saves at least as many dollars as Coupon B or C. What is $y - x$?

(A) 50 **(B)** 60 **(C)** 75 **(D)** 80 **(E)** 100

12. At the beginning of the school year, 50% of all students in Mr. Wells' math class answered "Yes" to the question "Do you love math", and 50% answered "No." At the end of the school year, 70% answered "Yes" and 30% answered "No." Altogether, x% of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of x?

(A) 0 **(B)** 20 **(C)** 40 **(D)** 60 **(E)** 80

13. What is the sum of all the solutions of $x = |2x - |60 - 2x||$?

(A) 32 **(B)** 60 **(C)** 92 **(D)** 120 **(E)** 124

14. The average of the numbers $1, 2, 3, \ldots, 98, 99$, and x is $100x$. What is x?

(A) $\frac{49}{101}$ **(B)** $\frac{50}{101}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{51}{101}$ **(E)** $\frac{50}{99}$

15. On a 50-question multiple choice math contest, students receive 4 points for a correct answer, 0 points for an answer left blank, and -1 point for an incorrect answer. Jesse's total score on the contest was 99. What is the maximum number of questions that Jesse could have answered correctly?

(A) 25 **(B)** 27 **(C)** 29 **(D)** 31 **(E)** 33

16. A square of side length 1 and a circle of radius $\sqrt{3}/3$ share the same center. What is the area inside the circle, but outside the square?

(A) $\frac{\pi}{3} - 1$ **(B)** $\frac{2\pi}{9} - \frac{\sqrt{3}}{3}$ **(C)** $\frac{\pi}{18}$ **(D)** $\frac{1}{4}$ **(E)** $2\pi/9$

17. Every high school in the city of Euclid sent a team of 3 students to a math contest. Each participant in the contest received a different score. Andrea's score was the median among all students, and hers was the highest score on her team. Andrea's teammates Beth and Carla placed 37th and 64th, respectively. How many schools are in the city?

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

18. Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1, 2, 3, \ldots, 2010\}$. What is the probability that $abc + ab + a$ is divisible by 3?

(A) $\frac{1}{3}$ **(B)** $\frac{29}{81}$ **(C)** $\frac{31}{81}$ **(D)** $\frac{11}{27}$ **(E)** $\frac{13}{27}$

19. A circle with center O has area 156π. Triangle ABC is equilateral, $\overline{BC}$ is a chord on the circle, $OA = 4\sqrt{3}$, and point O is outside $\triangle ABC$. What is the side length of $\triangle ABC$?

(A) $2\sqrt{3}$ **(B)** 6 **(C)** $4\sqrt{3}$ **(D)** 12 **(E)** 18

20. Two circles lie outside regular hexagon $ABCDEF$. The first is tangent to $\overline{AB}$, and the second is tangent to $\overline{DE}$. Both are tangent to lines BC and FA. What is the ratio of the area of the second circle to that of the first circle?

(A) 18 **(B)** 27 **(C)** 36 **(D)** 81 **(E)** 108

21. A palindrome between 1000 and 10,000 is chosen at random. What is the probability that it is divisible by 7?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{7}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{5}$

22. Seven distinct pieces of candy are to be distributed among three bags. The red bag and the blue bag must each receive at least one piece of candy; the white bag may remain empty. How many arrangements are possible?

(A) 1930 **(B)** 1931 **(C)** 1932 **(D)** 1933 **(E)** 1934

23. The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?

(A) 18 **(B)** 24 **(C)** 36 **(D)** 42 **(E)** 60

24. A high school basketball game between the Raiders and the Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?

(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

25. Let $a > 0$, and let $P(x)$ be a polynomial with integer coefficients such that

$$P(1) = P(3) = P(5) = P(7) = a, \text{ and}$$
$$P(2) = P(4) = P(6) = P(8) = -a.$$

What is the smallest possible value of a?

(A) 105 **(B)** 315 **(C)** 945 **(D)** 7! **(E)** 8!

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

American Mathematics Competitions
University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone 402-472-2257 | Fax 402-472-6087 | amcinfo@maa.org

The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

Dr. Leroy Wenstrom
lwenstrom@gmail.com

2010 AIME

The 28th annual AIME will be held on Tuesday, March 16, with the alternate on Wednesday, March 31. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 1% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 39th Annual USA Mathematical Olympiad (USAMO) on April 27 - 28, 2010. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: www.unl.edu/amc.