

Wednesday, FEBRUARY 21, 2007

58th Annual American Mathematics Contest 12

AMC 12 CONTEST B



THE MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions

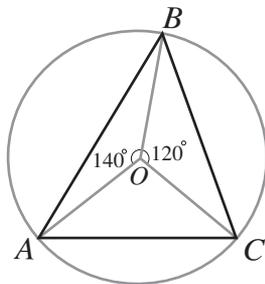
1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have 75 MINUTES to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 25th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 13, 2007 or Wednesday, March 28, 2007. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of the AMC 12 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules. After the contest period, permission to make copies of individual problems in paper or electronic form including posting on web-pages for educational use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear the copyright notice.

1. Isabella's house has 3 bedrooms. Each bedroom is 12 feet long, 10 feet wide, and 8 feet high. Isabella must paint the walls of all the bedrooms. Doorways and windows, which will not be painted, occupy 60 square feet in each bedroom. How many square feet of walls must be painted?
(A) 678 (B) 768 (C) 786 (D) 867 (E) 876
2. A college student drove his compact car 120 miles home for the weekend and averaged 30 miles per gallon. On the return trip the student drove his parents' SUV and averaged only 20 miles per gallon. What was the average gas mileage, in miles per gallon, for the round trip?
(A) 22 (B) 24 (C) 25 (D) 26 (E) 28
3. The point O is the center of the circle circumscribed about $\triangle ABC$, with $\angle BOC = 120^\circ$ and $\angle AOB = 140^\circ$, as shown. What is the degree measure of $\angle ABC$?



- (A) 35 (B) 40 (C) 45 (D) 50 (E) 60
4. At Frank's Fruit Market, 3 bananas cost as much as 2 apples, and 6 apples cost as much as 4 oranges. How many oranges cost as much as 18 bananas?
(A) 6 (B) 8 (C) 9 (D) 12 (E) 18
5. The 2007 AMC 12 contests will be scored by awarding 6 points for each correct response, 0 points for each incorrect response, and 1.5 points for each problem left unanswered. After looking over the 25 problems, Sarah has decided to attempt the first 22 and leave the last 3 unanswered. How many of the first 22 problems must she solve correctly in order to score at least 100 points?
(A) 13 (B) 14 (C) 15 (D) 16 (E) 17
6. Triangle ABC has side lengths $AB = 5$, $BC = 6$, and $AC = 7$. Two bugs start simultaneously from A and crawl along the sides of the triangle in opposite directions at the same speed. They meet at point D . What is BD ?
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

7. All sides of the convex pentagon $ABCDE$ are of equal length, and $\angle A = \angle B = 90^\circ$. What is the degree measure of $\angle E$?
- (A) 90 (B) 108 (C) 120 (D) 144 (E) 150
8. Tom's age is T years, which is also the sum of the ages of his three children. His age N years ago was twice the sum of their ages then. What is T/N ?
- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6
9. A function f has the property that $f(3x - 1) = x^2 + x + 1$ for all real numbers x . What is $f(5)$?
- (A) 7 (B) 13 (C) 31 (D) 111 (E) 211
10. Some boys and girls are having a car wash to raise money for a class trip to China. Initially 40% of the group are girls. Shortly thereafter two girls leave and two boys arrive, and then 30% of the group are girls. How many girls were initially in the group?
- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12
11. The angles of quadrilateral $ABCD$ satisfy $\angle A = 2\angle B = 3\angle C = 4\angle D$. What is the degree measure of $\angle A$, rounded to the nearest whole number?
- (A) 125 (B) 144 (C) 153 (D) 173 (E) 180
12. A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each of the juniors receive on the test?
- (A) 85 (B) 88 (C) 93 (D) 94 (E) 98
13. A traffic light runs repeatedly through the following cycle: green for 30 seconds, then yellow for 3 seconds, and then red for 30 seconds. Leah picks a random three-second time interval to watch the light. What is the probability that the color changes while she is watching?
- (A) $\frac{1}{63}$ (B) $\frac{1}{21}$ (C) $\frac{1}{10}$ (D) $\frac{1}{7}$ (E) $\frac{1}{3}$
14. Point P is inside equilateral $\triangle ABC$. Points Q , R , and S are the feet of the perpendiculars from P to \overline{AB} , \overline{BC} , and \overline{CA} , respectively. Given that $PQ = 1$, $PR = 2$, and $PS = 3$, what is AB ?
- (A) 4 (B) $3\sqrt{3}$ (C) 6 (D) $4\sqrt{3}$ (E) 9
15. The geometric series $a + ar + ar^2 + \dots$ has a sum of 7, and the terms involving odd powers of r have a sum of 3. What is $a + r$?
- (A) $\frac{4}{3}$ (B) $\frac{12}{7}$ (C) $\frac{3}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

16. Each face of a regular tetrahedron is painted either red, white, or blue. Two colorings are considered indistinguishable if two congruent tetrahedra with those colorings can be rotated so that their appearances are identical. How many distinguishable colorings are possible?
- (A) 15 (B) 18 (C) 27 (D) 54 (E) 81
17. If a is a nonzero integer and b is a positive number such that $ab^2 = \log_{10} b$, what is the median of the set $\{0, 1, a, b, 1/b\}$?
- (A) 0 (B) 1 (C) a (D) b (E) $\frac{1}{b}$
18. Let a , b , and c be digits with $a \neq 0$. The three-digit integer abc lies one third of the way from the square of a positive integer to the square of the next larger integer. The integer acb lies two thirds of the way between the same two squares. What is $a + b + c$?
- (A) 10 (B) 13 (C) 16 (D) 18 (E) 21
19. Rhombus $ABCD$, with side length 6, is rolled to form a cylinder of volume 6 by taping \overline{AB} to \overline{DC} . What is $\sin(\angle ABC)$?
- (A) $\frac{\pi}{9}$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$ (E) $\frac{\sqrt{3}}{2}$
20. The parallelogram bounded by the lines $y = ax + c$, $y = ax + d$, $y = bx + c$, and $y = bx + d$ has area 18. The parallelogram bounded by the lines $y = ax + c$, $y = ax - d$, $y = bx + c$, and $y = bx - d$ has area 72. Given that a , b , c , and d are positive integers, what is the smallest possible value of $a + b + c + d$?
- (A) 13 (B) 14 (C) 15 (D) 16 (E) 17
21. The first 2007 positive integers are each written in base 3. How many of these base-3 representations are palindromes? (A palindrome is a number that reads the same forward and backward.)
- (A) 100 (B) 101 (C) 102 (D) 103 (E) 104
22. Two particles move along the edges of equilateral $\triangle ABC$ in the direction

$$A \rightarrow B \rightarrow C \rightarrow A,$$

starting simultaneously and moving at the same speed. One starts at A , and the other starts at the midpoint of \overline{BC} . The midpoint of the line segment joining the two particles traces out a path that encloses a region R . What is the ratio of the area of R to the area of $\triangle ABC$?

- (A) $\frac{1}{16}$ (B) $\frac{1}{12}$ (C) $\frac{1}{9}$ (D) $\frac{1}{6}$ (E) $\frac{1}{4}$

23. How many non-congruent right triangles with positive integer leg lengths have areas that are numerically equal to 3 times their perimeters?

(A) 6 (B) 7 (C) 8 (D) 10 (E) 12

24. How many pairs of positive integers (a, b) are there such that $\gcd(a, b) = 1$ and

$$\frac{a}{b} + \frac{14b}{9a}$$

is an integer?

(A) 4 (B) 6 (C) 9 (D) 12 (E) infinitely many

25. Points A, B, C, D , and E are located in 3-dimensional space with $AB = BC = CD = DE = EA = 2$ and $\angle ABC = \angle CDE = \angle DEA = 90^\circ$. The plane of $\triangle ABC$ is parallel to \overline{DE} . What is the area of $\triangle BDE$?

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) $\sqrt{5}$ (E) $\sqrt{6}$

WRITE TO US!

Correspondence about the problems and solutions for this AMC 12 and orders for any of the publications listed below should be addressed to:

American Mathematics Competitions
University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone: 402-472-2257; Fax: 402-472-6087; email: amcinfo@unl.edu

The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

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2007 AIME

The AIME will be held on Tuesday, March 13, 2007, with the alternate on Wednesday, March 28, 2007. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above, or finish in the top 1% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on April 24 and 25, 2007. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site:
www.unl.edu/amc.

2007

AMC 12 – CONTEST B

DO NOT OPEN UNTIL

WEDNESDAY, February 21, 2007

****Administration On An Earlier Date Will Disqualify
Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE February 21.** Nothing is needed from inside this package until February 21.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC no later than 24 hours following the examination.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination during this period via copier, telephone, email, World Wide Web or media of any type is a violation of the competition rules.*

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